ALUPE UNIVERSITY COLLEGE

Bastion of Knowledge...

OFFICE OF THE DEPUTY PRINCIPAL

ACADEMICS, RESEARCH AND STUDENTS' AFFAIRS

UNIVERSITY EXAMINATIONS

2018 /2019 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS/SCI/B.ST)

COURSE CODE: **MAT 113**

COURSE TITLE: **DIFFERENTIAL CALCULUS**

DATE: 10TH DECEMBER, 2018   **TIME: 2.00 PM – 5.00 PM**

INSTRUCTION TO CANDIDATES

- **SEE INSIDE**

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MAT 113

REGULAR-MAIN EXAM
MAT 113: DIFFERENTIAL CALCULUS

STREAM: BEd. Sc/Art/Bus   DURATION: 3 Hours

INSTRUCTION TO CANDIDATES

i. Answer **ALL** questions from section A and any **THREE** from section B

ii. Do not write on this question paper

SECTION A: [31 MARKS]: ANSWER ALL QUESTIONS
Question One : [16 marks]

(a) Differentiate the following functions using the first principles rule

      i) $f(x) = \sqrt{x}$ [3 marks]

      ii) $f(x) = x^2$ [3 marks]

(b) Differentiate with respect to x, $f(x) = 5x^4 + 4x - \frac{1}{2x^2} + \frac{1}{\sqrt{x}} - 3$ [3 marks]

(c) Determine the rate of change of voltage, given $v(t) = 5t \sin 2t$ when $t = 0.2s$ [3 marks]

(d) Find a value of c such that the conclusion of the mean value theorem is satisfied for $f(x) = -2x^3 + 6x - 2$ on the interval $[-2, 2]$ [4 marks]

Question Two : [15 marks]

(a) State Rolles theorem   [3 marks]

(b) Evaluate the limit $\lim\limits_{x \to 0} \frac{\tan x}{x}$ [3 marks]

(c) Differentiate $f(x) = x^3 \cos(3x) \ln x$ [3 marks]

(d) Find the differential coefficient of

      i) $f(x) = \frac{4 \sin 5x}{5x^4}$ [3 marks]

      ii) $f(t) = \frac{te^{2t}}{2 \cos t}$ [3 marks]

SECTION B[39 MARKS]: ANSWER ANY THREE QUESTIONS
Question Three :[13 Marks]

(a) Find the derivatives of the functions;

      i) $f(x) = \sqrt{3x^2 + 4x - 1}$ [3 marks]

      ii) $f(x) = 3 \cos(5x^2 + 2)$ [4 marks]

(b) Show that the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y$ is satisfied when $y = xe^{2x}$ [6 marks]

Question Four: [13 Marks]

(a) Prove that $\lim\limits_{x \to 4} 3x - 5 = 7$ [2 marks]

(b) The displacement cm of the end of a stiff spring at time seconds is given by $s = ae^{-kt}sin(2ft)$. Determine the velocity at the end of the spring after $1s$ if $a = 2m$, $k = 0.9$ and $f = 5$ [6 marks]

(c) Determine the turning points of the curve $y = 4\sin x - 3\cos x$ in the range 0 to 2π radians, and distinguish them [5 marks]

Question Five: [13 Marks]

(a) Show that $\lim\limits_{x \to 2} \frac{x^2-4}{x-2} = 4$ [3 marks]

(b) A rectangular sheet of metal having dimension $20cm$ and $12cm$ has squares removed from each of the four corners and the sides bent upwards to form an open box. Determine the maximum possible volume of the box. [5 marks]

(c) Determine the equations of the tangent and normal to the curve $y = \frac{x^3}{5}$ at the point $(-1, -\frac{1}{5})$ [5 marks]

Question Six: [13 Marks]

(a) The parametric equations of a cycloid are $x = 4(\theta - \sin\theta)$, $y = 4(1 - 4\cos\theta)$. Determine $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ [7 marks]

(b) Given $2y^2 - 5x^4 - 2 - 7y^3 = 0$. Determine $\frac{dy}{dx}$ implicitly. [6 marks]

Question Seven: [13 Marks]

(a) Find the value of k that makes $f(x)$ continuous at $x = -3$.
$$f(x) = \begin{cases} x^2 - 4, & \text{if } x \leq -3; \\ k, & \text{if } x = -1. \end{cases}$$
[3 marks]

(b) Using logarithmic differentiation to differentiate $f(x) = \frac{(x-1)(x-2)^3}{(x-3)}$. [5 marks]

(c) Given that if $y = \tan^{-1}(\frac{\sin t}{\cos t - 1})$. Show that $\frac{dy}{dx} = \frac{1}{2}$. [5 marks]