ALUPE UNIVERSITY
OFFICE OF THE DEPUTY VICE CHANCELLOR

ACADEMICS, RESEARCH AND STUDENTS AFFAIRS

UNIVERSITY EXAMINATIONS

2024/2025 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER REGULAR MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED STATISTICS

COURSE CODE: STA 424

COURSE TITLE: STOCHASTIC PROCESSES

DATE: 9th April 2025 **TIME:** 08:00 – 11:00

INSTRUCTION TO CANDIDATES

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STA 424

REGULAR – MAIN EXAM

STA 424: STOCHASTIC PROCESSES

STREAM: BSC (Applied Statistics) **DURATION: 3 Hours**

INSTRUCTIONS TO CANDIDATES

i. *Answer ALL questions from section A and ANY THREE Questions in section B.*

ii. *All questions in section B carry Equal Marks.*

iii. *Do not write on the question paper.*

SECTION A (31 marks): Answer ALL questions

QUESTION ONE (16MKS)

a) Define the following terms

i Stochastic process (2 Marks)

ii Discrete – time (2 Marks)

iii State space, S (2 Marks)

iv Strict-Sense Stationary (SSS) (2 Marks)

b) Let N(t) be a Poisson process with intensity $\lambda=2$, and let $X_1, X_2, \cdots$ be the corresponding arrival times. Find the probability that the first arrival occurs after t=0.5, i.e., $P(X_1>0.5)$

(2 Marks)

c) If $\{a_k\} = \{0,0,0,1,1,1\ldots\}$ Find generating function of this sequence, that is, A(S)

(2 Marks)

d) Consider the Markov chain with three states, S={1,2,3}, that has the following transition

matrix $p=\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

If we know $P(X_1=1)=P(X_1=2)=\frac{1}{4}$, find $P(X_1=3, X_2=2, X_3=1)$. (4 Marks)

QUESTION TWO (15 Marks)

a) Let X have a Binomial distribution such that $P_k = \binom{n}{k} p^k q^{n-k}$ k = 0,1,2… . Find the probability generating function (p.g,f), mean and the variance (6 Marks)

b) Let X have a Poisson distribution such that $P_k = \frac{\lambda^k e^{-\lambda}}{k!}$ k = 0,1,2… . Find the probability generating function (p.g,f), mean and the variance (6 Marks)

c) If $\{a_k\} = \left\{\frac{1}{k!}\right\}$ $\forall k$ Find generating function of this sequence, that is, A(S) (3 Marks)

SECTION B (39 MARKS, CHOOSE ANY THREE QUESTIONS)

QUESTION THREE (13 MARKS)

a) A certain calculating machine uses only the digits 0 and 1. It is supposed to transmit one of these digits through several stages. However, at every stage, there is a probability p that the digit that enters this stage will be changed when it leaves and a probability q = 1 - p that it won't.

i. Form a Markov chain to represent the process of transmission by taking as states the digits 0 and 1. What is the matrix of transition probabilities? (1 marks)

ii. Now draw a tree and assign probabilities assuming that the process begins in state 0 and moves through two stages of transmission. (2 marks)

iii. What is the probability that the machine, after two stages, produces the digit 0 (i.e., the correct digit)? (2 marks)

b) Consider the knight's tour on a chess board: A knight selects one of the next positions at random independently of the past.

(i) Why is this process a Markov chain? (1mark)

(ii) What is the state space? (1mark)

(iii) Is it irreducible? Is it aperiodic? (2 marks)

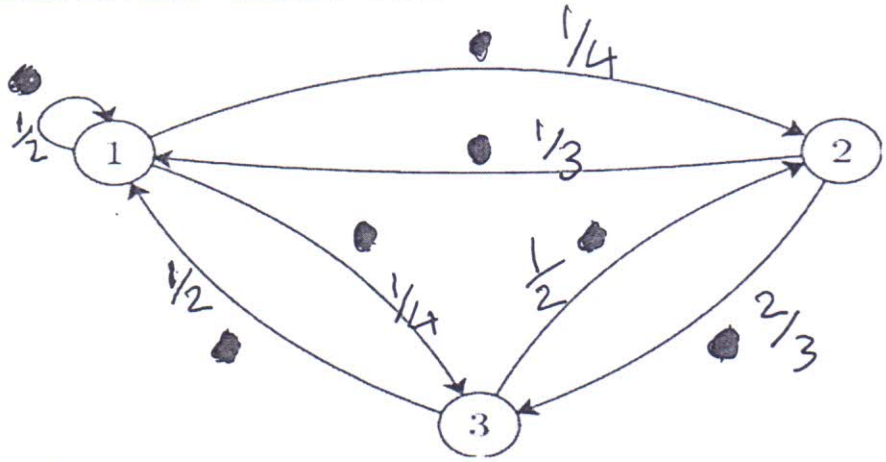
(iv) Find the stationary distribution. Give an interpretation of it: what does it mean, physically? (2mark)

(v) Which are the most likely states in steady-state? Which are the least likely ones? (2 marks)

QUESTION FOUR (13 Marks)

a) Let $\{a_k\} = \{1,1,2,3,5,8,13,21, \ldots.\}$ Be a Fibonacci series where $a_0 = 1$, $a_1 = 1$, $a_k = a_{k-1} + a_{k-2}$. Find a_{25} (7 Marks)

b) Consider the Markov chain shown below



i. Is this chain irreducible? (1 Mark)

ii. Is this chain aperiodic? (1 Mark)

iii. Find the stationary distribution for this chain. (2 Marks)

iv. Is the stationary distribution a limiting distribution for the chain? (2 Marks)

QUESTION FIVE (13 Marks)

a) $a_n + a_{n-1} - 16a_{n-2} + 20a_{n-3} = 0,$ $n \geq 3$ .Subject to initial value $a_0 = 0$ $a_1 = 1$ and $a_2 = -1$ Find A(s) and thus the value of a_k (7 Marks)

b) $a_n = 5_{n-1} - 6a_{n-2}$ $n \geq 2$. Subject to initial value $a_0 = 1$ $a_1 = 2$. Find A(s) and thus the value of a_k (6 Marks)

QUESTION SIX (13 Marks)

a) Student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to studying the next night as well. Find

(i) The transition matrix of this process. (2 Marks)

(ii) The transition matrix after 4 nights. (1 Marks)

(iii) Suppose that on the first night, the student toss affair die and studied if only if a 2 or 3 appeared. What is the probability that he didn't study in the fourth night. (2 Marks)

b) Suppose the process $\{X_{t}, : t \geq 0\}$ be a poisson process having rate λ=2 . Find:

i. $P\{X_2 = 4, X_5 = 12, X_9 = 16\}$ (4 Marks)

ii. $P\{X_{1.5} = 10, X_{3.5} = 18, X_5 = 20\}$ (4 Marks)

QUESTION SEVEN (13 Marks)

a) A salesman's area consists of three cities, A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. however, if he sells in either B or C, then the next day he is twice as likely to sell in city A as in the other city. Find

(i) The transition matrix of this process. (2 Marks)

(ii) In the long run, how often does he sell in each of the cities? (2 Marks)

b) Suppose we know that a receptionist receives an average of 15 phone calls per hour.

i) What is the probability that he will receive at least two calls between 8 and 8:12 am? (2 mark)

ii) If the receptionist absents for 10 minutes what is the probability that no call has been lost. (2 mark)

c) Consider the failures of a link in a communication network. Failures occur according to a Poisson process with rate 2.4 per day. Find:

i. Probability of time between failure greater than t days (2 mark)

ii. Probability of time between failure less than t days (1 mark)

iii. Probability of failures in t days (1 mark)

iv. Probability of 0 failures in next day. (1 mark)