ALUPE UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR

ACADEMICS, RESEARCH AND STUDENTS' AFFAIRS

UNIVERSITY EXAMINATIONS

2025/2026 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER REGULAR MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS & SCIENCE

COURSE CODE: MAT 418

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS

DATE: MONDAY, 15/12/2025 **TIME: 2-5PM**

INSTRUCTION TO CANDIDATES

- **SEE INSIDE**

THIS PAPER CONSISTS OF 3 PRINTED PAGESPLEASE TURN OVER

STREAM: CS **DURATION: 3 Hours**

INSTRUCTION TO CANDIDATES

i. *Answer **ALL** questions from **section A** and any **THREE** from **section B***
ii. *Do not write on the question paper.*

SECTION A, COMPULSARY (31 MARKS)

QUESTION ONE (16 MARKS)

a) Classify the following PDEs **(4 marks)**

i. $x^2\frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

ii. $\frac{\partial^2 u}{\partial x^2} + x^2\frac{\partial^2 u}{\partial y^2} = 0$

iii. $3u_{xx} + 2u_{xy} + 5u_{yy} + yu_y = 0$

b) Using D' Alembert's Method, solve the PDE $U_{tt} = 25U_{xx}$, where $t > 0, 0 \le x \le 1$

$$U(0, x) = \sin x$$

$$U_t(0, x) = x^2$$

(5 marks)

c) Solve $xzp + yzq = xy$ using Lagrange's method **(7 marks)**

QUESTION TWO (15 MARKS)

a) Solve $(mz-ny)\frac{\partial y}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx$ using method of multipliers. **(7 marks)**

b) Transform to its canonical form; $2u_{xx} - 4u_{xy} - 6u_{yy} + u_x = 0$ **(8 marks)**

SECTION B, ANSWER ANY THREE QUESTIONS (39 MARKS)

QUESTION THREE (13 MARKS)

a) Using method of characteristic, solve the first order linear equation; $x^2u_x + yu_y + xyu = 1$ **(3 marks)**

b) Find the solution of a string of length $L = 1$ that is parallel upward at the middle 50 that it reaches a height of 0.5 and satisfies the wave equation $U_{tt} = 9U_{xx}$
(10 marks)

QUESTION FOUR (13 MARKS)

a) State the maximum principle or Laplace equation **(2 marks)**

b) Consider the following BVP $u_{xx} + u_{yy} = 0$, $(x, y)\epsilon\Omega, 0 < x < 2, 0 < y < 1,\ x, y \in 2\Omega$, where x and y are on the boundary $\emptyset(x, y) = \sin \pi x + \cos \pi y$. Find the range of the values of the solution of $U(x, y)$. **(6 marks)**

c) Solve by method of grouping $2p + 3q = 1$ **(5 marks)**

QUESTION FIVE (13 MARKS)

a) Find the characteristic of ; $u_{xx} + u_{xy} + u_{yy} = 0$ **(3 marks)**

b) Solve the following IVP by method of separation of variables $U_{tt} = 36U_{xx}, t > 0$
$0 \leq x \leq 1$

$$U(0, x) = 0, \qquad U_t(0, x) = 4 \qquad 0 \leq x \leq 1$$

$$U(t, 0) = 0, \quad U(t, 1) = 0,$$

(10 marks)

QUESTION SIX (13 MARKS)

a) Solve by Charpit's Method; $xp + yq = pq$ **(7 marks)**

b) Find the general solution of $3u_{xx} + 3yu_{xy} + 3u_x = 0, y \neq 0.$ **(6 marks)**

QUESTION SEVEN (13 MARKS)

a) State the order and degree of the following PDEs; **(4 marks)**

i. $x^2 \frac{\partial^2 u}{\partial x^2} - 4xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$

ii. $\frac{\partial^4 u}{\partial x^2} + (x^2 \frac{\partial^2 u}{\partial y^2})^3 = 0$

b) Solve for $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y)$, by direct method **(5 marks)**

c) Solve $yzp + 2xq = xy$ by lagrange's method **(4 marks)**