ALUPE UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR

ACADEMICS, RESEARCH AND STUDENTS' AFFAIRS

UNIVERSITY EXAMINATIONS

2025/2026 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER REGULAR MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS & SCIENCE

COURSE CODE: MAT 404

COURSE TITLE: ~~ANALYTICAL~~ NUMERICAL METHODS

DATE: MONDAY, 15/12/2025 **TIME: 9AM-12PM**

INSTRUCTION TO CANDIDATES

- **SEE INSIDE**

THIS PAPER CONSISTS OF 3 PRINTED PAGESPLEASE TURN OVER

STREAM: CS **DURATION: 3 Hours**

INSTRUCTION TO CANDIDATES

i. *Answer **ALL** questions from **section A** and any **THREE** from **section B***
ii. *Do not write on the question paper.*

SECTION A. COMPULSARY (31 MARKS)

QUESTION ONE (15 MARKS)

a) Define a PDE and give an example of a linear PDE of order 2. **(2 marks)**

b) Expand $f(x) = x^3$ in the interval $(-\pi, \pi)$ as a Fourier series. **(6 marks)**

c) Solve the IVP using D-Alembert's method; $U_{tt} = 9U_{xx}$ $t > 0$ where initial conditions are;

$$u(0, x) = x^2 + 2x \qquad -\infty < x < \infty$$

$$u_t(0, x) = \cos 3x$$

(7 marks)

QUESTION TWO (16 MARKS)

a) Determine the Laplace transform of:

$2e^{3t}(4\cos 2t - 5\sin 2t)$ **(5 marks)**

b) Find the fourier cosine transform of $f(x) = e^{-mx}, m > 0$ **(7 marks)**

c) Verify the initial value theorem for the function whose Laplace transform is $(2t - 3)^2$

(4 marks)

SECTION B, ANSWER ANY THREE QUESTIONS (39 MARKS)

QUESTION THREE (13 MARKS)

a) Find $\Gamma(-\frac{5}{2})$ **(3 marks)**

b) Solve the following IVP by method of separation of variables

$U_{tt} = 36U_{xx},$ $t > 0, 0 \le x \le 1$, given that the conditions are,

$u(0, x) = 0, \quad u_t(0, x) = 4$

$u(t, 0) = 0, \quad u(t, 1) = 0$

(10 marks)

QUESTION FOUR (13 MARKS)

a) Solve $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + u$, using method of separation of variables **(5 marks)**

b) Expand e^x about $x = 0$ up to the term in x^4 using Taylor series. **(8 marks)**

QUESTION FIVE (13 MARKS)

a) Find $L\{-3 \sin 3t\}$ using the Laplace transform derivative for $F(t) = \cos 3t$ **(3 marks)**

b) Find $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$ **(5 marks)**

c) Find the fourier series to the function

$$f(x) = \begin{cases} 4 \; if -5 < x < 0 \\ 3 \; if \; 0 < x < 5 \end{cases} \qquad p = 10$$ **(5 marks)**

QUESTION SIX (13 MARKS)

a) Distinguish between linear and nonlinear PDE and give two examples in each case. **(5 marks)**

b) A tightly stretched string with fixed end points $x = 0$ and $x = l$ on the x-axis. At time

$t = 0$ it has a shape given by $f(x)$, $0 < x < l$, and is released from rest. Find the displacement of the string at any later time. Use $(\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2})$,

$y(x,t) = (A_1 \sin \lambda x + B_1 \cos \lambda x)(A_2 \sin \lambda at + B_2 \cos \lambda at)$ **(8 marks)**

QUESTION SEVEN (13 MARKS)

a) Show that spherical harmonics form an orthogonal set of functions. **(6 marks)**

b) An insulated rod of length *l* has its end A and B maintained $0^{\circ}C$ and $100^{\circ}C$ respectively until steady state condition Prevail. If B is suddenly reduced to $0^{\circ}C$ and maintained at $0^{\circ}C$. Find the temperature at a distance x from A at time t. **(7 marks)**