ALUPE UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR

ACADEMICS, RESEARCH AND STUDENTS' AFFAIRS

UNIVERSITY EXAMINATIONS

2024/2025 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER REGULAR MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE & COMPUTER SCIENCE

COURSE CODE: MAT 400

COURSE TITLE: ANALYTIC APPLIED MATHEMATICS

DATE: 7TH APRIL 2025 **TIME:** 2:00-5:00PM

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGESPLEASE TURN OVER

STREAM: CS DURATION: 3 Hours

INSTRUCTION TO CANDIDATES

i. *Answer **ALL** questions from **section A** and any **THREE** from **section B***
ii. *Do not write on the question paper.*

dimensional variables
dimensionless variables
$n-k=$
$5-3=2$

SECTION A, COMPULSARY (31 MARKS)

QUESTION ONE (16 MARKS)

$n=5$
$k=3$

a) A sphere of radius r falls through a fluid with viscosity μ, density ρ , and gravity g. Determine the dimensionless parameters governing the system. $5-3=2$ **(5 marks)**

b) Analyze the undamped duffing oscillator; $\ddot{x} + \dot{x} + x^3 = 0$ **(4 marks)**

c) Solve the eigenvalue problem; $y'' + \lambda y = 0 \quad 0 < x < L, y(0) = 0, y(L) = 0$ **(4 marks)**

d) Solve $y'' + 3y' + 2y = 0$, with $y(0) = 1$ and $y'(0) = 0$, using Laplace transforms **(3 marks)**

QUESTION TWO (15 MARKS)

a) A factory produces two products, x_1 and x_2, with profits of \$ 30 and \$ 40 per unit respectively. Each unit of x_1 requires 2 hours of labor and 3 hours of machine time and x_2 requires 4 hours of labor and 2 hours of machine time. The factory has 100 labor hours and 80 machine hours available. Find the production plan to maximize profit. **(6 marks)** $(0, 40)$

b) Find the Fourier transform of $(x) = e^{-a\|x\|}$, where $a > 0$ **(4 marks)**

c) Solve;

i. $y_{n+1} - 3y_n = 5, \; y(0) = 2$ **(2 marks)**

ii. $y_{n+2} - 4y_{n+1} + 3y_n = 0, \; y(0) = 1 , \; y(1) = 2$ **(3 marks)**

SECTION B, ANSWER ANY THREE QUESTIONS (39 MARKS)

QUESTION THREE (13 MARKS)

a) Consider the flow velocity V of a fluid overs a flat plate. The velocity depends on the plate Length L, fluid density ρ fluid viscosity μ, and the external force F. Derive the dimensionless groups using Buckingham π theorem. **(5 marks)**

b) Solve the population growth model using the logistic equation: $\frac{dp}{dt} = rp(1 - \frac{p}{k})$, where r is the growth rate and k is the carrying capacity. **(4 marks)**

c) Solve $x^2 - 2\epsilon x - 3 = 0$ for small ϵ. **(4 marks)**

QUESTION FOUR (13 MARKS)

a) Find the image of the upper-plane under the mapping $f(z) = z^2$ **(2 marks)**

b) Maximize the objective function $z = 3x + 2y$ subject to the constants;

1. $x + y \leq 4$
2. $x \geq 0$
3. $y \geq 0$

Find the function y(x) minimizing the arc length. **(4 marks)**

c) Find the function y(x) that etremizes the functional $J[Y] = \int_0^1 (y'^2 + y^2)\, dx$ subject to the following conditions $y(0) = 0$ and $y(1) = 1$ **(7 marks)**

QUESTION FIVE (13 MARKS)

a) Solve the integral equation $f(x) - \lambda \int_0^1 (xt) f(t)\, dt = x \quad 0 \le x \le 1$ **(4 marks)**

b) Solve the inhomogeneous boundary value problem using Green's function;
$y'' - y = f(x) \quad 0 < x < 1\, L, \; y(0) = 0, y(1) = 0.$ **(4 marks)**

c) Find the solution of the wave equations using D' Alembert's Formula

$$\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2} \text{ with boundary conditions; } u(x, 0) = e^{-x^2}, u_t(x, 0) = 0$$

(5 marks)

√QUESTION SIX (13 MARKS)

a) Approximate the integral $\int_0^2 x^3$ dx using Simpson's Rule with $n = 4$ **(6 marks)**

b) Solve the Van der Pol oscillator; $\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$ **(7 marks)**

QUESTION SEVEN (13 MARKS)

a) Solve the Sturm-Liouville equation with a singularity at $x = 0$;

$\frac{d}{dx}\left(x\frac{dy}{dx}\right) + \lambda x y = 0$ **(4 marks)**

b) Analyze the system; $\frac{dx}{dt} = y, \frac{dy}{dt} = -x$ **(4 marks)**

c) Find the solution to the following heat equation by use of method of separation of variables

$$\frac{\partial u}{\partial t} = \propto \frac{\partial^2 u}{\partial x^2} \text{ with boundary conditions; } u(0, t) = 0, u(L, t) = 0 \;\; u(x, 0) = f(x)$$ **(5 marks)**

b $\frac{dx}{dt} = y \cdots f(x,y)$

$\frac{dy}{dt} = -x \cdots g(x,y)$

Critical points

$y=0.\ x=0$

$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

eigen values

$\begin{bmatrix} -\lambda & 1 \\ -1 & \lambda \end{bmatrix} = 0$

$\lambda^2 + 1 = 0$

$\lambda = \pm 1$

$\lambda_1 = -1 \quad \lambda_2 = +1$

stable centre