ALUPE UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR

ACADEMICS, RESEARCH AND STUDENT'S AFFAIRS

UNIVERSITY EXAMINATIONS

2024/2025 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER REGULAR MAIN EXAMINATION

FOR THE DEGREE OF

BACHELOR OF EDUCATION SCIENCE.

COURSE CODE: MAT 221

COURSE TITLE: VECTOR ANALYSIS

DATE: WEDNESDAY, 9TH APRIL, 2025 TIME: 11—2PM

INSTRUCTIONS TO CANDIDATES

Answer **ALL** questions in section A and **TWO** questions in section B

DO NOT write on this question paper

TIME: 3 hours

SECTION A (COMPULSORY 34 MKS)

QUESTION 1 (18 Marks)

a. Given that $\vec{A} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} + 4\hat{k}$, find

(i) $\vec{A} \bullet \vec{B}$

(ii) $|3\vec{A} + 2\vec{B}|$

(iii) the angle between $\vec{A}$ and $\vec{B}$

(iv) $|\vec{A} \times \vec{B}|$ **(8 Marks)**

b. Find the area of the triangle whose vertices are A(2,1, -1) B(4,2, -2) C(3,0,0) and hence show that it is a right angled triangle. **(4 Marks)**

c. Find $\frac{d}{ds}\left[\vec{A} \bullet \frac{d\vec{B}}{ds} - \frac{d\vec{A}}{ds} \bullet \vec{B}\right]$ if $\vec{A}$ and $\vec{B}$ are differential functions of S

d. If $\vec{A} = x^2yz\hat{i} - 2xz^3\hat{j} + xz^2\hat{k}$ and $\vec{B} = 2z\hat{i} + y\hat{j} - x^2\hat{k}$, find $\frac{\partial^2}{dxdy}\left(\vec{A} \times \vec{B}\right)$ at the point $(2,0,3)$ **(6 Marks)**

QUESTION 2 (16 MARKS)

a. Find the velocity and acceleration of a particle which moves along the curve whose parametric equations are given by $x = 2\sin 3t, y = 2\cos 3t$ and $z = 8t$ at any time t. also find the magnitude of the velocity and acceleration. **(5 Marks)**

b. Show that curl grad $\phi = 0$ **(4 Marks)**

c. Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$ where r is a position vector of $p(x,y,z)$. (Where $r^2 = x^2 + y^2 + z^2$ and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$) **(5 Marks)**

d. Evaluate $\int_c \vec{A} \bullet d\vec{r}$ where c is the curve $y = 2x^2$ in the xy plane from the point (0,0) to the point (1,2) if $\vec{A} = 3xy\,\hat{i} - y^2\hat{j}$ **(2 Marks)**

SECTION B: CHOOSE ANY TWO QUESTIONS

QUESTION THREE (13 MARKS)

a. If the vector $\vec{A} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{B} = \sin t\,\hat{i} - \cos t\,\hat{j}$, find

i. $\frac{d}{dt}\left(\vec{A} \bullet \vec{B}\right)$

ii. $\frac{d}{dt}\left(\vec{A} \bullet \vec{A}\right)$

iii. $\frac{d}{dt}\left(\vec{A} \times \vec{B}\right)$

iv. $\frac{d}{dt}\left(\vec{B} \times \vec{A}\right)$ **(7 Marks)**

b. Show that $\frac{d}{dt}\left(\vec{A}.\vec{B}\right) = \vec{A}.\frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt}.\vec{B}$ **(3 marks)**

c. Find the unit vector tangent to the curve $x = e^{-t}$, $y = 3\cos 2t$ and $z = 3\sin t$ at $t = 0$ **(3 marks)**

QUESTION FOUR (13 MARKS)

a. Given ϕ (x,y,z) $=3x^2y - y^3z^2$, find grad ϕ at (1, -2, -1) **(4 marks)**

b. If $\mathbf{A} = 2yz\mathbf{i} - x^2y\mathbf{j} + xz^2\mathbf{k}$, $\mathbf{B} = x^2\mathbf{i} + yz\mathbf{j} - xy\mathbf{k}$ and $\phi = 2x^2yz^3$, find **(9 Marks)**

i. $(\mathbf{A}.\nabla)\phi$

ii. $\mathbf{A}.\nabla\phi$

iii. $(\mathbf{B}.\nabla)\mathbf{A}$

iv. $(\mathbf{A}\times\nabla)\phi$

v. $\mathbf{A}\times\nabla\phi$

QUESTION FIVE (13 MARKS)

a. Given $\phi = x^2yz + 4xz^2$ in the direction $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$, find the directional derivate at the point (1, -2, -1) and hence find the normal to the surface ϕ at this point. **(4 Marks)**

b. Verify the divergence theorem for A = $4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$ **(7 Marks)**

c. Define the terms

i. Solenoidal vector
ii. Irrotational vector **(2 Marks)**

QUESTION SIX (13 MARKS)

a. If $\vec{A} = \left(2x^2y - x^4\right)\hat{i} + \left(e^{xy} - y\sin 2x\right)\hat{j} + \cos y\hat{k}$, find

(i) $\frac{\partial^2 \vec{A}}{\partial x^2}$ (ii) $\frac{\partial^2 \vec{A}}{\partial y^2}$ (iii) $\frac{\partial^2 \vec{A}}{\partial x \partial y}$ **(3 Marks)**

b. Evaluate $\iiint_v \nabla \bullet \vec{F} dv$ where $\vec{F} = (2x^2-3z)\hat{i} - 2xy\,\hat{j} - 4x\,\hat{k}$ and v is the closed volume bounded by the planes x = 0, y = 0 and 2x + 2y + z = 4
(4 Marks)

c. State Green's theorem and hence verify it in the plane for

$\oint_c \left(3x^2 - 8y^2\right)dx + \left(4x - 6xy\right)dy$ where c is the boundary of the region defined by $y = \sqrt{x}, y = x^2$ **(6 Marks)**