ALUPE UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR
ACADEMICS, RESEARCH AND STUDENTS AFFAIRS

UNIVERSITY EXAMINATIONS
2023/2024 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER REGULAR MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE COMPUTER SCIENCE

COURSE CODE: **BCS 113**
COURSE TITLE: **LINEAR ALGEBRA**

DATE: 9th JAN 2024 **TIME:8:00-11:00AM**

INSTRUCTION TO CANDIDATES

- **SEE INSIDE**

REGULAR – MAIN EXAM

BCS 113: LINEAR ALGEBRA

STREAM: CS **DURATION: 3 Hours**

INSTRUCTION TO CANDIDATES

i. *Answer **ALL** questions from **section A** and any **TWO** from **section B***

ii. *Do not write on the question paper.*

SECTION A (31 MARKS): Answer all questions in this section.

SECTION A – COMPULSORY (34 MARKS)

QUESTION ONE(18 MARKS)

a) Let the vector space V=$\mathbb{R}^3$ and $W = \{(a,b,c); a+b+c=0\}$ i.e. W consists of those vectors each with the property that the sum of its components is zero. Is W a subspace of V. **(3 Marks)**

b) Use Gauss-Jordan elimination to solve the system of linear equations.

$$x_1 - x_2 + 3x_3 = 6$$
$$x_1 + 2x_2 + 4x_3 = 9$$
$$2x_1 + x_2 + 6x_3 = 11$$

(5 Marks)

c) Define the term linear transformation and hence determine if $T:\mathbb{R}^3 \to \mathbb{R}^3$ defined as $T(x_1, x_2) = (x_1 + x_2, 2x_1 - x_2)$ is a linear transformation. **(5 Marks)**

d) Reduce to echelon form and hence find the rank of the matrix

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 1 & 1 \\ 4 & 2 & 2 \end{pmatrix}$$

(5 Marks)

QUESTION TWO (16 MARKS)

a) Find

i. The inverse of the matrix A by first getting the adjoint.

$$A = \begin{pmatrix} 1 & 3 & -4 \\ 0 & -2 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

(8 Marks)

ii. Compute $(2A - A^T) + A^{-1}$ **(5 Marks)**

b) Distinguish between linear dependence and linear independence of vectors in a vector space and hence determine if the set $S = \{(1,1,-1),(2,-3,1),(8,-7,1)\}$ is linearly dependent in $\mathbb{R}^3$.

(3 Marks)

SECTION B – ANSWER ANY TWO (2) QUESTIONS

QUESTION THREE (13 MARKS)

a) Solve the system of linear equation below by using Cramer's rule

$$2x_1 + x_2 - 2x_3 = 10$$
$$3x_1 + 2x_2 + 2x_3 = 1$$
$$5x_1 + 4x_2 + 3x_3 = 4$$

(8 Marks)

b) Reduce to Reduced Row Echelon form and hence find the rank.

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 4 & 2 & 2 \\ 3 & 1 & 2 \end{pmatrix}$$

(5 Marks)

QUESTION FOUR (13 MARKS)

a) If A_{nxn} is a matrix, what do you understand by the following?

i) A is Symmetric

ii) A is Anti-symmetric

iii) The transpose of A

(6 Marks)

b) i) If a matrix A_{nxn} is invertible, then show that the inverse is unique. **(3 Marks)**

ii) Find the inverse of the matrix by first appending an identity matrix to the right hand side of A and reducing the left hand side of $[A|I]$ to identity matrix.

$$A = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(4 Marks)

QUESTION FIVE (13 MARKS)

a) Solve by getting the inverse

$$x_1 - 2x_2 + x_3 = 7$$
$$2x_1 - x_2 + 4x_3 = 17$$
$$3x_1 - 2x_2 + 2x_3 = 17$$

(7 Marks)

b) Show whether or not the matrix $A = \begin{pmatrix} 2 & 0 \\ 3 & 0 \end{pmatrix}$ is invertible **(2 Marks)**

c) Let the vector space V=$\mathbb{R}^4$ and $S = \{(1,-2,0,3), (2,3,0,-1), (2,-1,2,1)\}$. Determine if $(3,9,-4,-2) \in L(S)$ (where $L(S)$ is the set spanned by S). **(4 Marks)**

QUESTION SIX (13 MARKS)

a) If $S = \{v_1, v_2, ..., v_n\}$ is a basis for a vector space V then show that every set with more than n vectors is linearly dependent. **(5 Marks)**

b) Find the basis and dimension of the solution space for the equations

$$2x_1 + 2x_2 - x_3 + x_5 = 0$$
$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$
$$x_1 + x_2 - 2x_3 - x_5 = 0$$
$$x_3 + x_4 + x_5 = 0$$

(5 Marks)

c) For the vectors $v_1 = (1,1,-1), v_2 = (4,0,1), v_3 = (3,-1,2)$

i) Find a basis for the subspace spanned by these vectors.

ii) Write the remaining vectors as a linear combination as the vectors in the basis.

(5 Marks)

QUESTION SEVEN (13 MARKS)

a) Given that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined as

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2x_1 + x_2 - x_3 \\ -x_3 + x_2 \\ x_1 - x_2 - x_3 \end{pmatrix}$$, find the matrix of T with respect to standard basis.

(3 Marks)

b) For which rational numbers a does the following system have

i) No solution

ii) Exactly one solution

iii) Infinitely many solutions

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (a^2 - 14)z = a + 2$$

(4 Marks)

c) The linear transformation $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} -y \\ -x \end{bmatrix}$ defines a reflection in the line $y = -x$.

Determine the standard matrix of this transformation. Find the kernel and the image of $\begin{bmatrix} x \\ y \end{bmatrix}$. **(6 Marks)**