ALUPE UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS
2023 /2024 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS

COURSE CODE: **MAT 110**
COURSE TITLE: **BASIC CALCULUS**

DATE: 5th December 2023 **TIME: 9:00AM-12:00PM**

INSTRUCTION TO CANDIDATES

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MAT 110: BASIC CALCULUS

STREAM: BSc (CS&ASC) **DURATION: 3 Hours**

INSTRUCTION TO CANDIDATES

i. *Answer **ALL** questions from **section A** and any **THREE** from **section B***

ii. *Do not write on the question paper.*

SECTION A (31 MARKS): Answer all questions in this section.

QUESTION ONE (16 MARKS)

a) Use the definition of the derivative, $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, to compute the derivative of $f(x)=2x^2-3x+6$. (3 Marks)

b) Evaluate each of the following limits

i) $\lim_{x\to 1}\frac{x^2+7x-8}{x-1}$ (2 Marks)

ii) $\lim_{x\to \infty}\frac{\sin x}{x^2}$ (2 Marks)

iii) $\lim_{x\to -1^-}\frac{x^3}{(x+1)^2}$ (2 Marks)

iv) $\lim_{x\to 0}\frac{\sin 3x}{\frac{x}{3}}$ (2 Marks)

c) Determine the equation of the tangent line to the semicircle with parametric equations $x=\cos t, y=\sin t, at\ t={}^{\pi}/_{4}$ (2mks)

d) Find an equation of the tangent line to the graph of the equation $x^2+9xy+y^2=36$ at the point (0, 6). (3 Marks)

QUESTION TWO (15 MARKS)

a) Find the derivative of differentiable functions

i) $y=\sin(x^3)$ (3 Marks)

ii) $y=(x^2)\cdot f(x)$ (3 Marks)

iii) $y=5\sin^4(x^3-3x^2)$. (3 Marks)

b) Find the maximum value of $f(x)=x^3+2x^2-4x$ on the interval $[-3, 1]$. (4 Marks)

c) Differentiate $y=x^x$ (2 Marks)

SECTION B (39 MARKS)[ANSWER ANY THREE QUESTIONS]

QUESTION THREE (13 MARKS)

a) Differentiate each of the following functions

i) $y = \dfrac{(x^2+4)^5}{(1-2x^2)^3}$ (4 Marks)

ii) $y = 3e^{2x} + 10x^3 \ln x$ (2 Marks)

b) Show that $f(x) = \frac{1}{2}x - \sqrt{x}$ satisfies the hypothesis of Rolle's Theorem on [0, 4], and find all values of c in (0, 4)that satisfy the conclusion of the theorem (4 Marks)

c) An object is shot upwards from ground level with an initial velocity of 2 meters per second; it is subject only to the force of gravity (no air resistance). Find its maximum altitude and the time at which it hits the ground. (3 Marks)

QUESTION FOUR (13 MARKS)

a) Find the value of *k* that makes the function *g* continuous at $x = 0$. (3 Marks)

$$g(x) = \begin{cases} x-2, & \text{if } x \le 0 \\ k(3-2x) & \text{if } x > 0 \end{cases}$$

b) A spherical balloon is being blown up at a rate of 100 cm^3/min. At what rate is its radius *r* changing when *r* is 4 cm? (4 Marks)

c) Find the maximum value and minimum value of $f(x) = (x-3)^{2/3}$ on [0, 4]. (4 Marks)

d) If $\frac{dV}{dt} = -32, V(0) = 64$, what is $V(t)$? (2 Marks)

QUESTION FIVE(13 MARKS)

a) Let $f(x) = 4x^2 + x$

i) Find the slope of the tangent to the curve when x = 1 using the definition of a limit. (3 Marks)

ii) Find the equation of the tangent line to the curve at the point (1, 5). (3 Marks)

b) Determine the maximum area: Alex uses 100 m of fence to enclose two adjacent rectangular fields (5 Marks)

c) Evaluate $sin^{-1}\left(\frac{1}{2}\right)$ (2 Marks)

QUESTION SIX (13 MARKS)

a) If $f(x)=2\sqrt{x}\ln x$ and $g(x)=\ln(\ln x)$, find $f'(x)$ and $g'(x)$ (4 Marks)

b) Determine whether $g(x)=\begin{cases}\dfrac{x^2-6x+9}{x-3}, & x\neq 3\\ o, & x=3\end{cases}$ is continuous at $x=3$ (4 Marks)

c) For which values of c does $\lim\limits_{x\to\infty}\dfrac{13}{cx^2+41}$ exist (3 Marks)

d) Find the first two derivatives of $R(t) = 3t^2 + 8t^{1/2} + e^t$ (2 Marks)

QUESTION SEVEN (13 MARKS)

a) Differentiate both sides of the equation

i) $x^3+y^3=4$ (3 Marks)

ii) $(x-y)^2=x+y-1$ (3 Marks)

iii) $y=\sin(3x+4y)$ (3 Marks)

b) When $f(x)=x^2-2x+1$ show that $f'(x)=0$ has at least one root in the interval $0<x<2$ using Rolle's Theorem and find the exact root. (4 Marks)