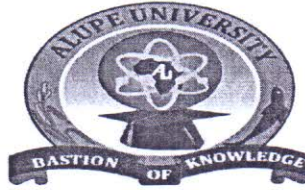


MAT 212



ALUPE UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR ACADEMICS,

RESEARCH AND STUDENT AFFAIRS

UNIVERSITY
EXAMINATIONS 2023/2024
ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER REGULAR MAIN EXAMINATION

**FOR THE DEGREE OF BACHELOR OF
EDUCATION SCIENCE/ARTS**

COURSE CODE: MAT 212

COURSE TITLE: LINEAR ALGEBRA I

DATE: 19TH DECEMBER 2023

TIME: 9.00AM – 12.00PM

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 4 PRINTED PAGES

PLEASE TURN OVER

INSTRUCTION TO CANDIDATES

- i. Answer ALL Questions from Section A and any THREE from Section B
- ii. Do not Write on the Question Paper
- iii. Answers Should be Comprehensive, Informative and Neat

SECTION A (31 MARKS): Answer ALL Questions in this Section

QUESTION ONE (16 MARKS)

a) Define the terms:

- (i) Subspace (1 mark)
- (ii) Augmented Matrix (1 mark)
- (iii) Span (1 mark)

b) Two linearly dependent vectors in \mathbb{R}^4 : Prove that vectors $v_1 = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 3 \end{pmatrix}$ and

$v_2 = \begin{pmatrix} -6 \\ 3 \\ 0 \\ -9 \end{pmatrix}$ are linearly dependent (4 marks)

c) Show that H is a subset of the vector space M_{22} with the standard operations of matrix addition and scalar multiplication (4 marks)

$$v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, v_2 = \begin{pmatrix} -4 \\ 1 \\ 5 \end{pmatrix}, v_3 = \begin{pmatrix} -5 \\ 8 \\ 19 \end{pmatrix}$$

d) If v_1, v_2, v_3 are linearly independent, show that the zero vector can be written as linear combination of v_1, v_2 and v_3 (3 marks)

e) Prove that: $\alpha 0 = 0$ (2 marks)

QUESTION TWO (15 MARKS)

a) A basis for a subspace \mathbb{R}^3 . Find a basis for a set of vectors lying on the plane:

$$\pi = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 2x - y + 3z = 0 \right\} \quad (4 \text{ marks})$$

b) Show that B is the inverse of A where $A = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix}$ (4 marks)

c) Show that the set $S = \{(1 \ 2 \ 3), (0 \ 1 \ 2), (-2 \ 0 \ 1)\}$ spans \mathbb{R}^3 (3 marks)

d) Show a linear transformation from \mathbb{R}^2 to \mathbb{R}^3 (4 marks)

SECTION B (39 MARKS): Answer any THREE Questions from this Section

QUESTION THREE (13 MARKS)

a) Prove that the set of all polynomials of degree 2 or less with the operations of addition and scalar multiplication is a vector space (3 marks)

b) Determine whether the three vectors in \mathfrak{R}^3 are linearly dependent or independent:

$$\begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \text{ and } \begin{pmatrix} 11 \\ -6 \\ 12 \end{pmatrix} \quad (3 \text{ marks})$$

c) Show that the following set is a basis of \mathfrak{R}^3 :

$$S = \{(1 \ 0 \ 0), (0 \ 1 \ 0), (0 \ 0 \ 1)\} \quad (3 \text{ marks})$$

d) Compute A^{-2} in two ways and show that the results are equal given that:

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix} \quad (4 \text{ marks})$$

QUESTION FOUR (13 MARKS)

a) Span S is a subspace of V if $S = \{v_1, v_2, \dots, v_k\}$ is a set of vectors in a vector space V , then show span S is a subspace of V (4 marks)

b) Prove that in \mathfrak{R}^3 , $\begin{pmatrix} -7 \\ 7 \\ 7 \end{pmatrix}$ is a linear combination of $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}$ (4 marks)

c) Use Gaussian elimination to solve:

$$b + c - 2d = -3$$

$$a + 2b - c = 2$$

$$2a + 4b + c - 3d = -2$$

$$a - 4b - 7c - d = -19$$

(5 marks)

QUESTION FIVE (13 MARKS)

a) Prove that if $\{u_1, u_2, \dots, u_m\}$ and $\{v_1, v_2, \dots, v_n\}$ are bases for the vector space V then $m = n$ (5 marks)

b) Determine the dimension of \mathfrak{R}^3 of: $W = \{(2b, b, 0)\} : b \text{ is a real number}$ (3 marks)

c) Find $(AB)^{-1}$ for the matrices: $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 3 \end{pmatrix}$ (5 marks)