ALUPE UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR ACADEMICS,

RESEARCH AND STUDENT AFFAIRS

UNIVERSITY EXAMINATIONS 2023/2024 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER REGULAR MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS

COURSE CODE: MAT 212

COURSE TITLE: LINEAR ALGEBRA I

DATE: 19TH DECEMBER 2023 **TIME:** 9.00AM – 12.00PM

INSTRUCTION TO CANDIDATES

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INSTRUCTION TO CANDIDATES

i. *Answer ALL Questions from Section A and any THREE from Section B*
ii. *Do not Write on the Question Paper*
iii. *Answers Should be Comprehensive, Informative and Neat*

SECTION A (31 MARKS): Answer ALL Questions in this Section

QUESTION ONE (16 MARKS)

a) Define the terms:
- **(i)** Subspace **(1 mark)**
- **(ii)** Augmented Matrix **(1 mark)**
- **(iii)** Span **(1 mark)**

b) Two linearly dependent vectors in $\Re^4$: Prove that vectors $v_1 = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 3 \end{pmatrix}$ and $v_2 = \begin{pmatrix} -6 \\ 3 \\ 0 \\ -9 \end{pmatrix}$ are linearly dependent **(4 marks)**

c) Show that H is a subset of the vector space M_{22} with the standard operations of matrix addition and scalar multiplication **(4 marks)**

d) If $v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $v_2 = \begin{pmatrix} -4 \\ 1 \\ 5 \end{pmatrix}$ and $v_3 = \begin{pmatrix} -5 \\ 8 \\ 19 \end{pmatrix}$, show that the zero vector can be written as linear combination of v_1 , v_2 and v_3 **(3 marks)**

e) Prove that: $\alpha 0 = 0$ **(2 marks)**

QUESTION TWO (15 MARKS)

a) A basis for a subspace $\Re^3$. Find a basis for a set of vectors lying on the plane:

$$\pi = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 2x - y + 3z = 0 \right\}$$ **(4 marks)**

b) Show that B is the inverse of A where $A = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix}$ **(4 marks)**

c) Show that the set $S = \{(1 \quad 2 \quad 3), (0 \quad 1 \quad 2), (-2 \quad 0 \quad 1)\}$ spans $\Re^3$ **(3 marks)**

d) Show a linear transformation from $\Re^2$ to $\Re^3$ **(4 marks)**

SECTION B (39 MARKS): Answer any THREE Questions from this Section

QUESTION THREE (13 MARKS)

a) Prove that the set of all polynomials of degree 2 or less with the operations of addition and scalar multiplication is a vector space **(3 marks)**

b) Determine whether the three vectors in $\Re^3$ are linearly dependent or independent:

$$\begin{pmatrix}1\\-3\\0\end{pmatrix},\begin{pmatrix}3\\0\\4\end{pmatrix} \text{ and } \begin{pmatrix}11\\-6\\12\end{pmatrix}$$ **(3 marks)**

c) Show that the following set is a basis of $\Re^3$:

$S=\{(1\quad 0\quad 0),(0\quad 1\quad 0),(0\quad 0\quad 1)\}$ **(3 marks)**

d) Compute A^{-2} in two ways and show that the results are equal given that:

$$A=\begin{pmatrix}1 & 1\\2 & 4\end{pmatrix}$$ **(4 marks)**

QUESTION FOUR (13 MARKS)

a) Span S is a subspace of V if $S=\{v_1,v_2,\ldots,v_k\}$ is a set of vectors in a vector space V, then show span S is a subspace of V **(4 marks)**

b) Prove that in $\Re^3$, $\begin{pmatrix}-7\\7\\7\end{pmatrix}$ is a linear combination of $\begin{pmatrix}-1\\2\\4\end{pmatrix}$ and $\begin{pmatrix}5\\-3\\1\end{pmatrix}$ **(4 marks)**

c) Use Gaussian elimination to solve:

$$b+c-2d=-3$$
$$a+2b-c=2$$
$$2a+4b+c-3d=-2$$
$$a-4b-7c-d=-19$$

(5 marks)

QUESTION FIVE (13 MARKS)

a) Prove that if $\{u_1,u_2,\ldots,u_m\}$ and $\{v_1,v_2,\ldots,v_n\}$ are bases for the vector space V then $m=n$ **(5 marks)**

b) Determine the dimension of $\Re^3$ of: $W=\{(2b,b,0)\}: b$ is a real number **(3 marks)**

c) Find $(AB)^{-1}$ for the matrices: $A=\begin{pmatrix}1 & 3 & 3\\1 & 4 & 3\\1 & 3 & 4\end{pmatrix}$ and $B=\begin{pmatrix}1 & 2 & 3\\1 & 3 & 3\\2 & 4 & 3\end{pmatrix}$ **(5 marks)**