ALUPE UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR

ACADEMICS, RESEARCH AND STUDENTS' AFFAIRS

UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER REGULAR MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS/SCIENCE

COURSE CODE: MAT 214

COURSE TITLE: VECTOR ANALYSIS

DATE: 25^{TH} **APRIL 2024** **TIME: 2.00PM – 5.00PM**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGES **PLEASE TURN OVER**

MAT 214 (130)

STREAM: BED (Arts/Science) **DURATION: 3 Hours**

INSTRUCTIONS TO CANDIDATES

i. *Answer ALL Questions from* ***section A*** *and any* ***THREE*** *from* ***section B.***

ii. *Do not write on the question paper.*

SECTION A (31 Marks)

Answer ALL questions in this section

Question One (16 Marks)

a) Find the unit vector parallel to the resultant of vectors, $\vec{r}_1 = 2\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}$ and $\vec{r}_2 = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$. (4 Marks)

b) If $\phi = xy^2z$ evaluate $\int_C \phi d\vec{r}$ along the curve C having parametric equations $x = 3t, y = 2t^2, z = t^3$ between $P_1(0,0,0)$ and $P_2(3,2,1)$. (5 Marks)

c) A particle moves along a curve whose parametric equations are $x = e^{-t}, y = 2\cos 3t, z = 2\sin 3t$ where t is the time,

i) Determine its velocity and acceleration at any time, t. (3 Marks)

ii) Find the magnitudes of the velocity and acceleration at $t = 0$. (4 Marks)

Question Two (15 Marks)

a) Find the unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$. (5 Marks)

b) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\boldsymbol{i} - 5z\boldsymbol{j} + 10x\boldsymbol{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$. (5 Marks)

c) Determine the values of a so that $\vec{A} = 3\boldsymbol{i} - 6\boldsymbol{j} + 2\boldsymbol{k}$ makes with the coordinate axis. (5 Marks)

SECTION A (39 Marks)

Answer any THREE questions from this section.

Question Three (13 Marks)

a) Find the angles that the vector $\vec{r} = 3\boldsymbol{i} - 6\boldsymbol{j} - 2\boldsymbol{k}$ makes with the coordinate axes. (6 Marks)

b) Determine the vector having intial point $P(x_1, y_1, z_1)$ and terminal point $Q(x_2, y_2, z_2)$ and find its magnitude. (4 Marks)

c) If $\emptyset = 2xy^2z + x^2y$, evaluate $\int_C \emptyset d\boldsymbol{r}$ where C is the curve $x = t, y = t^2, z = t^3$ from $t = 0$ to $t = 1$. (3 Marks)

Question Four (13 Marks)

a) Verify Stokes' theorem for $\vec{\boldsymbol{F}} = (2x - y)\boldsymbol{i} - yz^2\boldsymbol{j} - y^2z\boldsymbol{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (5 Marks)

b) Evaluate $\iint_S \vec{\boldsymbol{F}} \circ \vec{\boldsymbol{n}} dS$ over the entire surface of the region above the xy −plane bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 4$, if $\vec{\boldsymbol{F}} = 4xz\boldsymbol{i} + yz^2\boldsymbol{j} + 3z\boldsymbol{k}$. (5 Marks)

c) If $\vec{\boldsymbol{A}} = (y^2 - x^2z^2)\boldsymbol{i} + (x^2 + y^2)\boldsymbol{j} - x^2yz\boldsymbol{k}$, determine $curl\vec{\boldsymbol{A}}$ at the point $(1,3,-2)$. (3 Marks)

Question Five (13 Marks)

a) If $\vec{\boldsymbol{A}} = t^2\boldsymbol{i} - t\boldsymbol{j} + (2t + 1)\boldsymbol{k}$ and $\vec{\boldsymbol{B}} = (2t - 3)\boldsymbol{i} + \boldsymbol{j} - t\boldsymbol{k}$ and $\emptyset = 2x^2yz^2$, find

i) $\frac{d}{dt}(\vec{\boldsymbol{A}} \circ \vec{\boldsymbol{B}})$ (2 Marks)

ii) $\frac{d}{dt}(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}})$ (2 Marks)

b) Show that $\nabla\phi$ is a vector perpendicular to the surface $\phi(x, y, z) = C$ where C is a constant. (4 Marks)

c) Define the following terms:

i) Solenoid Vector (1 Mark)

ii) Rectifying plane (1 Mark)

d) Find the area of the triangle having vertices at $P(1,3,2), Q(2,-1,1)$ and $R(-1,2,3)$. (4 Marks)

Question Six (13 Marks)

a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the direction of $2\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$. (4 Marks)

b) Find the total work done in moving a particle in a force field given by $\vec{\boldsymbol{F}} = \boldsymbol{3xyi} - 5z\boldsymbol{j} + \boldsymbol{10xk}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^2$ from $t = 1$ to $t = 2$. (4 Marks)

c) Find the area of the upper cap cut from the sphere $x^2 + y^2 + z^2 = 2$ by the cylinder $x^2 + y^2 = 1$. (5 Marks)

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SECTION A (39 Marks)

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Question Three (13 Marks)

a) Find the angles that the vector $\vec{r} = 3i - 6j - 2k$ makes with the coordinate axes. (6 Marks)

b) Determine the vector having intial point $P(x_1, y_1, z_1)$ and terminal point $Q(x_2, y_2, z_2)$ and find its magnitude. (4 Marks)

Question Seven (13 Marks)

a) Find the area of the surface cut from the bottom of the paraboloid $z = x^2 + y^2$ by the plane $z = 1$. (4 Marks)

b) Represent $\vec{F} = z\boldsymbol{i} - 2x\boldsymbol{j} + y\boldsymbol{k}$ in cylindrical coordinates. Thus determine F_ρ, F_ϕ, F_z. (6 Marks)

c) Find the constant a such that the vectors $2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $3\boldsymbol{i} + a\boldsymbol{j} + 5\boldsymbol{k}$ are coplanar. (3 Marks)

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