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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2021 /2022 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS AND SCIENCE AND BACHELOR OF SCIENCE APPLIED STATISTICS AND COMPUTING

COURSE CODE: MAT 310/311E

COURSE TITLE: ADVANCED REAL ANALYSIS/REAL ANALYSIS II

DATE: 3RD FEB 2022 TIME: 9.00AM – 12.00NOON

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 2 PRINTED PAGES PLEASE TURN OVER

INSTRUCTIONS TO CANDIDATES

i. *Answer ALL Questions from **section A** and any **THREE** from **section B**.*

ii. *Do not write on the question paper.*

SECTION A (31 Marks)

Answer ALL questions from this section.

Question One (16 Marks)

a) Prove that the constant function $f(x) = 1$ on $[0,1]$ is Riemann integrable and $\int_0^1 1dx = 1$.

(6 Marks)

b) Define the following terms

i) Continuous function

ii) Uniform continuity

iii) Riemann integral (6 Marks)

f) Show that the improper integral $\int_0^\infty \frac{\sin(x^2)(x+2)}{x^3+1} dx$ converges. (4 Marks)

Question Two (15 Marks)

a) Let $f: [0, \infty) \to \mathbb{R}$ be a function defined by $f(x) = \sqrt{x}$. Show that f is continuous on $[0, \infty)$.

(5 Marks)

b) Show that the sequence $f_n(x) = x^n$ converges pointwise on $[0,1]$ but not uniformly on $[0,1]$.

(5 Marks)

c) Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. Show that $\lim_{x \to c} f(x) = \lim_{x \to c} x^2 = c^2$. (5 Marks)

SECTION B (39 Marks)

Answer any THREE questions from this section.

Question Three (13 Marks)

a) Let $\{f_n\}$ be a sequence of continuous functions $f_n: S \to \mathbb{R}$ converging uniformly to $f: S \to \mathbb{R}$. Prove that f is continuous. (8 Marks)

b) Prove that if $f: I \to \mathbb{R}$ be differentiable at $c \in I$, then it is continuous at c. (5 Marks)

Question Four (13 Marks)

a) Prove that a continuous function $f: [a, b] \to \mathbb{R}$ on a compact interval is Riemann integrable.

(10 Marks)

b) Prove that the function $f(x) = \sqrt{x}$ is differentiable for $x > 0$. (3 Marks)

Question Five (13 Marks)

a) Show that a monotonic increasing function $f: [a, b] \to \mathbb{R}$ on a compact interval is Riemann integrable. (9 Marks)

b) Prove that series $\sum_{n=1}^{\infty} nx^n$ converges to $\frac{1}{(1-x)^2}$ on $(-1,1)$. (4 Marks)

Question Six (13 Marks)

a) Let $f(x) = \frac{1}{|x|+1}$. Show that $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to -\infty} f(x) = 0$. (4 Marks)

b) Let $f: [a, b] \to \mathbb{R}$ be a continuous function. Show that f is uniformly continuous. (9 Marks)

Question Seven (13 Marks)

a) Let $f_n: S \to \mathbb{R}$ be bounded functions. Prove that $\{f_n\}$ is Cauchy in the uniform norm if and only if there exists an $f: S \to \mathbb{R}$ and $\{f_n\}$ converges uniformly to f. (8 Marks)

b) Let $f: (0,1) \to \mathbb{R}$ be a function defined by $f(x) = \frac{1}{x}$. Show that it is not uniformly continuous.

(5 Marks)

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