ALUPE UNIVERSITY COLLEGE
...Bastion of Knowledge....
P. O.Box 845-50400 Busia(K)
principal@auc.ac.ke
Tel: +254 741 217 185
+254 736 044 469
off Busia-Malaba road

OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2021 /2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS AND SCIENCE

COURSE CODE: **MAT 415E**

COURSE TITLE: **DIFFERENTIAL GEOMETRY**

DATE: 9TH JUNE, 2022 **TIME: 1400 – 1700 HRS**

INSTRUCTION TO CANDIDATES

- **SEE INSIDE**

THIS PAPER CONSISTS OF 3 PRINTED PAGES **PLEASE TURN OVER**

REGULAR – MAIN EXAM

MAT 415E DIFFERENTIAL GEOMETRY

STREAM: BED (Arts/Science) **DURATION: 3 Hours**

INSTRUCTIONS TO CANDIDATES

i. *Answer ALL Questions from* ***section A*** *and any* ***THREE*** *from* ***section B****.*

ii. *Do not write on the question paper.*

SECTION A (31 Marks)

Answer ALL questions in this section.

Question One (16 Marks)

a) Show that $x = te_1 + (t^2+1)e_2 + (t-1)e_3$ is a regular representation for all t and find the projections onto the x_1x_2 and x_1x_3 planes. (6 Marks)

b) Compute the length of the arc $x = 3(\cos 2t)e_1 + 3(\sinh 2t)e_2 + 6e_3$, $0 \le t \le \pi$. (4 Marks)

c) Find the equations of the tangent line and normal plane to the curve $x = (1+t)e_1 - t^2e_2 + (1+t^3)e_3$ at $t = 1$. (4 Marks)

d) What is a regular curve? (2 Marks)

Question Two (15 Marks)

a) Find the curvature vector k and curvature $|k|$ on the curve $x = te_1 + \frac{1}{2}t^2e_2 + \frac{1}{3}t^3e_3$ at the point $t = 1$. (5 Marks)

b) Show that the mapping $x = u^2e_1 + uve_2 + v^2e_3$ is a coordinate patch of class C^∞ on the first quadrant $u > 0, v > 0$. (5 Marks)

c) Let $x = x(u,v)$ and $x = x^*(\theta, \phi)$ be coordinate patches on a simple surface S defined on open sets U and U^* respectively and having overlapping images G and G^* on S. Let W and W^* be the subsets of U and U^* respectively which map onto $G \cap G^*$. Show that W and W^* are open sets in their respective parameter planes. (5 Marks)

SECTION B (39 Marks)

Answer any THREE questions.

Question Three (13 Marks)

a) Prove that $f(x)$ is continuous at x_0 if $f(x)$ is differentiable at x_0. (6 Marks)

b) Find the derivative of $f(x) = (x_1 \sin x_2)e_1 + (x_2 \sin x_1)e_2$ in the direction $u_0 = \left(\frac{1}{\sqrt{5}}\right)e_1 + \left(\frac{2}{\sqrt{5}}\right)e_2$ at $x_0 = \left(\frac{\pi}{2}\right)e_1 + \left(\frac{\pi}{4}\right)e_2$. (4 Marks)

c) If $\lim_{x \to x_0} f(x) = f(x_0)$, show that f is continuous at x_0. (3 Marks)

Question Four (13 Marks)

a) Determine the rank and image of the linear mapping. (4 Marks)

$$y_1 = 2x_1 + x_2$$
$$y_2 = -4x_1 - 2x_2$$
$$y_3 = -2x_1 - x_2$$

b) Show that the points on the helix $x = a(\cos t)e_1 + a(\sin t)e_2 + bte_3$ at which the osculating plane pass through a fixed point are confined to a plane. (5 Marks)

c) Find the intersection of the x_1x_2 plane and the tangent lines to the helix $x = (\cos t)e_1 + (\sin t)e_2 + te_3$ $(t > 0)$. (4 Marks)

Question Five (13 Marks)

a) Show that $x = \frac{1}{2}\left(s + \sqrt{s^2+1}\right)e_1 + \frac{1}{2}\left(s + \sqrt{s^2+1}\right)^{-1} e_2 + \frac{1}{2}\sqrt{2}\left(\log(s + \sqrt{s^2+1})\right)e_3$ is a natural representation, that is $\left|\frac{dx}{ds}\right| = 1$. (6 Marks)

b) Show that the arc $x = t^2 e_1 + \sin t e_2$, $0 \leq t \leq {}^{\pi}/_{2}$ is rectifiable. (7 Marks)

Question Six (13 Marks)

a) If f and g are continuous at x_0, show that $f.g$ is continuous at x_0. (7 Marks)

b) Let $y = (x_1^2 + x_2^2)e_1 + x_1x_2^2e_2$, $x = (u_1 \cos u_2)e_1 + (u_1 \sin u_2)e_2$. Find the derivatives $\frac{\partial y_1}{\partial u_1}$ and $\frac{\partial y_1}{\partial u_2}$. (6 Marks)

Question Seven (13 Marks)

a) Find the following derivatives of $f(x) = (x_1^2 + x_2^2)g_1 + x_1e^{x_2}g_2 + x_2e^{x_1}g_3$ at the point $x_0 = e_1 - e_2$.

i) $\frac{\partial^2 f}{\partial x_1 \partial x_2}$ (3 Marks)

ii) $\frac{\partial^3 f}{\partial^2 x_1 \partial x_2}$ (2 Marks)

iii) $D^2_{v_0u_0}f$ in the directions $u_0 = (e_1 + e_2)$ and $v_0 = (e_1 - 2e_2)$. (4 Marks)

b) Show that the function $f(x) = (e^{x_1}\cos x_2)e_1 + (e^{x_1}\sin x_2)e_2$ satisfies the conditions of the inverse function theorem on E^2 but is not $1-1$ on E^2. (4 Marks)

****