OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2021 /2022 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION

COURSE CODE: MAT 113

COURSE TITLE: DIFFERENTIAL CALCULUS

DATE: 21ST JANNUARY, 2022   TIME: 0900 – 1200 HRS

INSTRUCTION TO CANDIDATES

- SEE INSIDE

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REGULAR - MAIN EXAM

MAT 113: DIFFERENTIAL CALCULUS

STREAM: EDS & EDA   DURATION: 3 Hours

INSTRUCTIONS TO CANDIDATES

i. Answer **All** questions from **Section A** and any **Three** from **Section B**

ii. Do not write on the question paper.

SECTION A (31 MARKS)

Question One (16 Marks)

(a) State the largest possible subset of real numbers $\mathbb{R}$ which would be a suitable domain and describe the corresponding range of the function $f(x) = \sqrt{10 - x}$.   (3 Marks)

(b) Given that $f(x) = 10 - x$ and $g(x) = x^2 + 2$,

    (i) Find $g'(x)$.   (1 Mark)

    (ii) Show that $f \circ g(x) \neq g \circ f(x)$.   (3 Marks)

(c) Consider the function

$$f(x) = \begin{cases} x^{\frac{1}{3}}, & \text{if } x < 8 \\ 2, & \text{if } x = 8 \\ \sqrt{x - 4}, & \text{if } x > 8. \end{cases}$$

Describe the continuity at $x = 8$.   (3 Marks)

(d) Find the following limits if they exist

    (i) $\lim\limits_{x \to 5} \frac{x^2 - 4x - 5}{x - 5}$.   (2 Marks)

    (ii) $\lim\limits_{x \to \infty} \frac{5x + 1}{10 + 2x}$.   (2 Marks)

(e) Differentiate $y = \ln(x^2 + 4)$.   (2 Marks)

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Question Two (15 Marks)

(a) Define the following terms

   (i) Limit of a function.   (1 Marks)

   (ii) Implicit function.   (1 Marks)

(b) Differentiate

   (i) $y = (2x - x^2)^3 \sqrt{x^2}$.   (3 Marks)

(c) Find the slope of the tangent of the curve $x^2 + \frac{x}{y} + y^2 = 7$ at the point $(1, 2)$.   (3 Marks)

(d) State the mean value theorem of differential calculus.   (3 Marks)

(e) Find the value of c prescribed by the law of mean value theorem given that $f(x) = 3x^2 + 4x - 3$.   (4 Marks)

SECTION B (39 MARKS)

Question Three (13 Marks)

(a) Describe the continuity of the function $f(x) = \frac{x^3 - x^2 - 9x - 9}{x^2 - 9}$.   (3 Marks)

(b) Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = t^3 - 3t^2 + 5$, $y = 2t - 7$, at $t = 2$.   (4 Marks)

(c) The profit of XYZ, a manufacturing concern, is defined by the function $P = 5000 + 200x - 2x^2$, where x is the amount of raw materials needed to produce commodity A. Determine

   (i) the amount of raw material that will maximize the profit.   (3 Marks)

   (ii) the maximum profit.   (3 Marks)

Question Four (13 Marks)

(a) Find the equation of the tangent line to the curve $y = \frac{3x^2 - 1}{x}$ at the point $(1, 2)$.   (4 Marks)

(b) Find the coordinates of the points on the graph of $(x - 2y - 1)^2 + (x + y)^2 = 16$ when the tangent is horizontal.   (6 Marks)

(c) Find the derivative of $f(x) = \frac{x^2 + 3x - 4}{2x + 1}$.   (3 Marks)

Question Five (13 Marks)

(a) Differentiate

   (i) $y = \frac{\cos 2x}{\sin 3x}$.   (3 Marks)

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(ii) $y = \cos(2x^2 - 1)^3$. (3 Marks)

(b) State the Rolle's theorem of differential calculus. (2 Marks)

(c) Suppose that $f(x) = x^{\frac{1}{2}} - x^{\frac{3}{2}}$ is continuous and differentiable on $(0, 1)$. Find the number c that satisfies the Rolle's theorem. (5 Marks)

Question Six (13 Marks)

(a) Find the derivative of the following

   (i) $y = e^{5x^2 - 2x + 7}$. (3 Marks)

   (ii) $y = \frac{\ln x}{x}$. (3 Marks)

(b) List the members of the following sets

   (i) $A = \{x | x \in \mathbb{N}\}$ & x is a multiple of 3. (2 Marks)

   (ii) $B = \{x | x \in \mathbb{Z}\}$ & $0 < x \le 6$. (2 Marks)

(c) Use L'Hospitals rule to obtain $\lim\limits_{t \to 1} \frac{5t^4 - 4t^2 - 1}{10 - t + 9t^3}$. (3 Marks)

Question Seven (13 Marks)

(a) Define a function. (1 Mark)

(b) A particle moves along a path with a position that can be determined by the function $x(t) = 4t^2 + e^t$. Determine its velocity and acceleration at time $t = 3$ seconds. (4 Marks)

(c) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ if $x = t^3 + 1$ and $y = t^2 + 2$, where $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ are all functions of x. (6 Marks)

(d) Find f' of $f(x) = (x^3 + 4)^5$. (2 Marks)