UNIVERSITY EXAMINATIONS
2021 /2022 ACADEMIC YEAR
FIRST YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE CS/ASC

COURSE CODE: MAT 110
COURSE TITLE: BASIC CALCULUS

DATE: 20th JANUARY, 2022   TIME: 2:00-5:00PM

INSTRUCTION TO CANDIDATES
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MAT 110: BASIC CALCULUS

STREAM: BSc (CS&ASC)   **DURATION: 3 Hours**

INSTRUCTION TO CANDIDATES

i. Answer *ALL* questions from *section A* and any *THREE* from *section B*

ii. Do not write on the question paper.

SECTION A (31 marks)

QUESTION ONE (15 Marks)

(a) Given that $f(x) = x^2 - 1$ and $g(x) = 3x + 5$ find $f \circ g(x)$ and its domain   (3Marks)

(b) Evaluate the following limits

   (i) $\lim\limits_{x \to \infty} \dfrac{\sqrt{4x^2 + 1}}{2x + 3}$ (3 Marks)

   (ii) $\lim\limits_{x \to \frac{3}{2}} \dfrac{4x^2 - 9}{2x - 3}$ (3 Marks)

   (iii) $\lim\limits_{x \to 0} \dfrac{\tan x}{x}$ (2 Marks)

(c) Find the area of the region bounded by the curve $y = x^2 - 4$, the $x-axis$ and the lines $x = 1$ and $x = 3$ (2 Marks)

(d) Determine the derivative of $y = e^{4x}$ (2 Marks)

QUESTION TWO (16 Marks)

(a) Use the definition of the derivative to determine the derivative of $f(x) = x^2$ (2 Marks)

(b) Find $\dfrac{dy}{dx}$ if $x = t^3 - t$ and $y = 4 - t^2$ (3 Marks)

(c) Given that $2x^3 - 3y^2 = 8$. Compute $\dfrac{d^2y}{dx^2}$ (5 Marks)

(d) Determine whether the function g defined by

$$g(x)=\begin{cases}3+x & if & x\le 1\\ 3-x & if & x>1\end{cases}$$

Is continuous at the point $x=1$ (3 Marks)

e) Prove that $\lim\limits_{x\to 4} 3x-5=7$ (2 Marks)

SECTION B (39 MARKS)

QUESTION THREE (13 Marks)

(a) Determine the derivative of $y = x^3 \ln(2x+5)$ (5 Marks)

(b) Show that $\dfrac{d(a^x)}{dx} = a^x \ln a$ (4 Marks)

(c) Find the derivative of the function $y = x^x$ (4 Marks)

QUESTION FOUR (13 Marks)

(a) Show that $\lim\limits_{x\to 2} \dfrac{x^2-4}{x-2} = 4$ (4 Marks)

(b) Compute $\dfrac{dy}{dx}$ for $y = \sin\left(\dfrac{2x}{x+1}\right)$ (3 Marks)

(c) Compute the following limits

    (i) $\lim\limits_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$ (3 Marks)

    (ii) $\lim\limits_{x\to 9} \dfrac{2\sqrt{x}-6}{x-9}$ (3 Marks)

QUESTION FIVE (13 Marks)

(a) Differentiate from first principles the function $f(x) = \dfrac{x}{x-1}$ (5Marks)

(b) Find the derivative of $y = e^{-x} \sin 3x$ (5Marks)

(c) Show that the derivative of $\sec x$ is $\sec x \tan x$ (3Marks)

QUESTION SIX (13 Marks)

(a) Find the derivative with respect to x of $x^3 y + 2y^4 - x^4 = 0$ at the point $(1,2)$. (4 Marks)

(b) Determine the second derivative with respect to x of $x = t^2$ and $y = t^3$ (5 Marks)

(c) Determine the derivative of $y = \ln(2x+5)(x^3 - 3)$ at the point $x = 1$ (4 Marks)

QUESTION SEVEN (13 Marks)

(a) A tank is in the form of an inverted cone having an altitude of $16\,ft$ and a base of radius $4\,ft$. Water is flowing into the tank at the rate of $2\,ft^3/min$. How fast is the water level rising when the water is $5\,ft$ deep? (5 Marks)

(b) Find the area of the Largest rectangle having a perimeter of $200\,ft$ (4 Marks)

(c) Find an equation of the line tangent to the curve $16x^4 + y^4 = 32$ at the point $(1,2)$. (4 Marks)