



OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2020 /2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER EXAMINATION

FOR THE DEGREE OF BACHELOR OF
EDUCATION SCIENCE

MAIN EXAM

COURSE CODE: MAT 104

COURSE TITLE: BASIC MATHEMATICS AND
ANALYTIC GEOMETRY

DATE: 18/02/2021

TIME: 0900 – 1200 HRS

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF PRINTED PAGES

PLEASE TURN OVER

MAT 104
REGULAR - MAIN EXAM

MAT 104: BASIC MATHEMATICS AND ANALYTIC GEOMETRY

STREAM: BSC (CS & ACS)

DURATION: 3 Hours

INSTRUCTIONS TO CANDIDATES

- i. Answer All questions from Section A and any Three from Section B
- ii. Do not write on the question paper.

SECTION A (31 MARKS)

Question One (16 Marks)

- (a) Define the following terms
 - (i) Gradient (1 Mark)
 - (ii) Radian measure of an angle. (1 Mark)
- (b) Show that $1 + \tan^2 \theta = \sec^2 \theta$. (4 Marks)
- (c) Find the remainder when the polynomial $P(x) = x^4 + 3x^3 + 2x + 3$ is divided by $x - 3$. (3 Marks)
- (d) Find the equations of the lines that have the tangents to the curve $y = x^3 - 6x + 8$ and parallel to the line $y = 6x - 2$. (4 Marks)
- (e) Find the equation of the common cord of two circles given by $x^2 + y^2 - 4x - 2y + 1 = 0$ and $x^2 + y^2 + 4x - 6y + 10 = 0$. (3 Marks)

Question Two (15 Marks)

- (a) Calculate the smallest angle between the minute hand and an hour hand at 6:12. (3 Marks)
- (b) Express the following in rectangular coordinates $r^2 = 1 + \sin \theta$. (2 Marks)
- (c) Find the distance of the point $(0, 4)$ from the line $2x + y - 7 = 0$. (3 Marks)

- (d) Sketch and describe the ellipse $9x^2 + 25y^2 = 225$. (4 Marks)
- (e) A sub-committee of six including a chairperson is to be chosen from the main committee of twelve. If the chairperson is to be chosen from the main committee, in how many ways can the sub-committee be chosen? (3 Marks)

SECTION B (39 MARKS)

Question Three (13 Marks)

- (a) Define the term angle. (1 Mark)
- (b) Solve $3 \cos 2\theta - \sin \theta + 2 = 0$ for $0 \leq \theta \leq 2\pi$. (4 Marks)
- (c) Show that $\cosh^2 x - \sinh^2 x = 1$. (3 Marks)
- (d) Express $\frac{5}{6}\pi$ in degrees. (2 Marks)
- (e) A new railway line is to be constructed parallel to the existing road, but 14m away. Find the equation of the railway line if a road passes through the point (6, 4) with equation $3x - 4y = 7$. (3 Marks)

Question Four (13 Marks)

- (a) Define the term hyperbola and write its standard equation when centered at the origin. (2 Marks)
- (b) Write an equation of the tangent at the point (2, 2) to the curve $x^2 - 2xy + y^2 + 2x + y = 0$. (4 Marks)
- (c) Find the radius and the centre of the circle with the equation $4x^2 + 4y^2 + 12x - 16y = 11$. (4 marks)
- (d) Express the given equation in polar coordinates $(x + y)^2 = x - y$. (3 Marks)

Question Five (13 Marks)

Consider the hyperbola that has the equation $9x^2 - 4y^2 + 18x + 18y = 31$. Find its centre, foci, length, latus rectum, equation of the asymptotes and its vertices. (13 marks)

Question Six (13 Marks)

- (a) Evaluate ${}^n C_2 = 1$. (2 Marks)
- (b) If ${}^n C_r = 56$ and ${}^n P_r = 6720$, find r . (3 Marks)
- (c) Find the equation of the line perpendicular to the line $4x + 5y + 7 = 0$ and passing through the point (6, -5). (3 Marks)
- (d) Find the points of intersection of the curve $r = 1$ and $r = 2 \sin \theta$. (2 Marks)

- (e) Solve $\sin^2 \theta = \frac{1}{4}$ for $0 \leq \theta \leq 2\pi$. (3 Marks)

Question Seven (13 Marks)

- (a) Identify the conic with the equation

$$r = \frac{18}{3 - 6 \cos \theta},$$

then find the vertices. (3 Marks)

- (b) Determine whether the circles $x^2 + y^2 - 2x + 4y + 2 = 0$ and $x^2 + y^2 + 8x + 2y - 22 = 0$ are orthogonal. (5 Marks)
- (c) Find the equations of tangent and normal to the curve $y = 3x^2 - 8x + 5$ at the point where $x = 2$. (5 Marks)