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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2020 /2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

MAIN EXAM

COURSE CODE: MAT 418

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS I

DATE: 16/03/2021 **TIME: 1400 – 1700 HRS**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

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MAT 418

RUGULAR – MAIN EXAMINATION

MAT 418: PARTIAL DIFFERENTIAL EQUATIONS I

STREAM: **BED SCI/ARTS** **TIME:** 3 HRS

EXAMINATION SESSION: MARCH **YEAR:** 2020/2021

INSTRUCTIONS TO CANDIDATES

(i) *Answer all questions in section A (Compulsory)*
(ii) *Answer any other THREE questions in section B*
(iii) *Answers should be comprehensive, informative and neat.*

SECTION A (31 MARKS)

Question One (16 Marks)

a). What is a partial differential equation? **(1 Mark)**

b). Find the general solution to the equation $2u_x + 3u_y + 8u = 0$. **(5 Marks)**

c). Find the equation of the tangent plane to the hyperboloid $4x^2 - 9y^2 - 9z^2 - 36 = 0$ at point $(3,3,2)$. **(4 Marks)**

d). Solve the differential equation $z = px + qy + p^2 + q^2$ where $p = z_x$ and $q = z_y$. **(3 Marks)**

e). Find the Monge's form of the equation of the surface $x = u + v, y = u - v, z = 4uv$. **(3 Marks)**

Question Two (15 Marks)

a). Form a p.d.e whose solution is $\phi(x^2 + y^2 + z^2, xyz) = 0$ **(3 Marks)**

b). Find the equation of the normal plane to the curve at the intersection of the surfaces $z_1 = xy + x$ and $z_2 = 2y$ at point $(1,0,1)$. **(3 Marks)**

c). Find the equation of the normal line to the surface $x = u, y = v, z = \frac{1}{2}(u^2 - v^2)$ at $p_0(3,1,2)$. **(4 Marks)**

d). Find the integral surface of the set of equations

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$$ **(5 Marks)**

SECTION B (39 MARKS)

Question Three (13 Marks)

a). Find the surface which orthogonally intersects the surface of the system $zx + zy = c(z + 1)$ which passes through the circle $x^2 + y^2 = 1, z = 1$. **(8 Marks)**

b). Find the integrating factor hence solve the equation

$$2x^2ydx + (x^3 + 2xy)dy = 0 \qquad \textbf{(5 Marks)}$$

Question Four (13 Marks)

a). Find the integral surface $\phi(c_1, c_2) = 0$ of the quasi-linear p.d.e.

$$xu_x + yu_y + xy(z^2 + 1) = 0 \qquad \textbf{(7 Marks)}$$

b). Show that the surface $F(x, y, z) = x^2 + 4y^2 - 4z^2 - 4 = 0$ and $G(x, y, z) = x^2 + y^2 + z^2 - 6x - 6y + 2z = 0$ are tangent at point $P(2,11)$. **(6 Marks)**

Question Five (13 Marks)

a). Find the integral curves of the equation

$$\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx} \qquad \textbf{(5 Marks)}$$

b). Find the auxiliary equations for orthogonal trajectories on the conicoid $z(x + y) = 1$ of conics which its cut by the system of planes $x - y + z = k$ where k is a parameter. **(8 Marks)**

Question Six (13 Marks)

a). Find the integral curves of the equation at $x^2 + y^2 = 1$ when $z = 1$.

$$\frac{dx}{x(y - z)} = \frac{dy}{y(z - x)} = \frac{dz}{z(x - y)} \qquad \textbf{(8 Marks)}$$

b). Find the equation of the tangent plane to the curve $x = \cos t, y = 3 + \sin 2t, z = 1 + \cos 3t$ at $t = \frac{\pi}{2}$. **(5 Marks)**

Question Seven (13 Marks)

a). Find the solution to the system

$$\frac{dx}{y + z} = \frac{dy}{y} = \frac{dz}{x - y} \qquad \textbf{(7 Marks)}$$

b). Find the Complete solution of the equation $z = p^2 - q^2$ **(6 Marks)**